EXAMPLE 4 Sketch intersections of planes

Sketch two planes that intersect in a line.

Solution

STEP 1 Draw a vertical plane. Shade the plane.

STEP 2 Draw a second plane that is horizontal. Shade this plane a different color. Use dashed lines to show where one plane is hidden.

STEP 3 Draw the line of intersection.

✓ GUIDED PRACTICE for Examples 3 and 4

4. Sketch two different lines that intersect a plane at the same point.

Use the diagram at the right.

5. Name the intersection of $\overrightarrow{PQ}$ and line $k$.
6. Name the intersection of plane $A$ and plane $B$.
7. Name the intersection of line $k$ and plane $A$.

1.1 EXERCISES

SKILL PRACTICE

1. VOCABULARY Write in words what each of the following symbols means.
   a. $Q$
   b. $MN$
   c. $\overrightarrow{ST}$
   d. $\overrightarrow{FG}$

2. ★ WRITING Compare collinear points and coplanar points. Are collinear points also coplanar? Are coplanar points also collinear? Explain.

NAMING POINTS, LINES, AND PLANES In Exercises 3–7, use the diagram.

3. Give two other names for $\overrightarrow{WQ}$.
4. Give another name for plane $V$.
5. Name three points that are collinear. Then name a fourth point that is not collinear with these three points.
6. Name a point that is not coplanar with $R$, $S$, and $T$.
7. ★ WRITING Is point $W$ coplanar with points $Q$ and $R$? Explain.
EXAMPLE 2 on p. 4 for Exs. 8–13

Naming segments and rays In Exercises 8–12, use the diagram.

8. What is another name for \( \overrightarrow{ZY} \)?
9. Name all rays with endpoint \( V \).
10. Name two pairs of opposite rays.
11. Give another name for \( \overrightarrow{WW} \).

12. Error analysis A student says that \( \overrightarrow{WW} \) and \( \overrightarrow{Z} \) are opposite rays because they have the same endpoint. Describe the error.

13. ★ Multiple choice Which statement about the diagram at the right is true?

- A) \( A, B, \) and \( C \) are collinear.
- B) \( C, D, E, \) and \( G \) are coplanar.
- C) \( B \) lies on \( \overrightarrow{GE} \).
- D) \( \overrightarrow{EF} \) and \( \overrightarrow{ED} \) are opposite rays.

Examples 3 and 4 on pp. 4–5 for Exs. 14–23

Sketching intersections Sketch the figure described.

14. Three lines that lie in a plane and intersect at one point

15. One line that lies in a plane, and one line that does not lie in the plane

16. ★ Multiple choice Line \( \overrightarrow{AB} \) and line \( \overrightarrow{CD} \) intersect at point \( E \). Which of the following are opposite rays?

- A) \( \overrightarrow{EC} \) and \( \overrightarrow{ED} \)
- B) \( \overrightarrow{CE} \) and \( \overrightarrow{DE} \)
- C) \( \overrightarrow{AB} \) and \( \overrightarrow{BA} \)
- D) \( \overrightarrow{AE} \) and \( \overrightarrow{BE} \)

Reading diagrams In Exercises 17–22, use the diagram at the right.

17. Name the intersection of \( \overrightarrow{PR} \) and \( \overrightarrow{HR} \).

18. Name the intersection of plane \( EFG \) and plane \( FGS \).

19. Name the intersection of plane \( PQS \) and plane \( HGS \).

20. Are points \( P, Q, \) and \( F \) collinear? Are they coplanar?

21. Are points \( P \) and \( G \) collinear? Are they coplanar?

22. Name three planes that intersect at point \( E \).

23. Sketching planes Sketch plane \( J \) intersecting plane \( K \). Then draw a line \( l \) in plane \( J \) that intersects plane \( K \) at a single point.

24. Naming rays Name 10 different rays in the diagram at the right. Then name 2 pairs of opposite rays.

25. Sketching Draw three noncollinear points \( J, K, \) and \( L \). Sketch \( \overrightarrow{JK} \) and add a point \( M \) on \( JK \). Then sketch \( \overrightarrow{ML} \).

26. Sketching Draw two points \( P \) and \( Q \). Then sketch \( \overrightarrow{PQ} \). Add a point \( R \) on the ray so that \( Q \) is between \( P \) and \( R \).
**ALGEBRA** In Exercises 27–32, you are given an equation of a line and a point. Use substitution to determine whether the point is on the line.

27. \( y = x - 4; A(5, 1) \) 
28. \( y = x + 1; A(1, 0) \) 
29. \( y = 3x + 4; A(7, 1) \)

30. \( y = 4x + 2; A(1, 6) \) 
31. \( y = 3x - 2; A(-1, -5) \) 
32. \( y = -2x + 8; A(-4, 0) \)

**GRAPHING** Graph the inequality on a number line. Tell whether the graph is a segment, a ray or rays, a point, or a line.

33. \( x \leq 3 \) 
34. \( x \geq -4 \) 
35. \(-7 \leq x \leq 4 \)

36. \( x \geq 5 \) or \( x \leq -2 \) 
37. \( x \geq -1 \) or \( x \leq 5 \) 
38. \( |x| \leq 0 \)

39. **CHALLENGE** Tell whether each of the following situations involving three planes is possible. If a situation is possible, make a sketch.
   a. None of the three planes intersect.
   b. The three planes intersect in one line.
   c. The three planes intersect in one point.
   d. Two planes do not intersect. The third plane intersects the other two.
   e. Exactly two planes intersect. The third plane does not intersect the other two.

**PROBLEM SOLVING**

**EVERYDAY INTERSECTIONS** What kind of geometric intersection does the photograph suggest?

40. [Image]
41. [Image]
42. [Image]

43. **SHORT RESPONSE** Explain why a four-legged table may rock from side to side even if the floor is level. Would a three-legged table on the same level floor rock from side to side? Why or why not?

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44. **SURVEYING** A surveying instrument is placed on a tripod. The tripod has three legs whose lengths can be adjusted.
   a. When the tripod is sitting on a level surface, are the tips of the legs coplanar?
   b. Suppose the tripod is used on a sloping surface. The length of each leg is adjusted so that the base of the surveying instrument is level with the horizon. Are the tips of the legs coplanar? Explain.

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1.2 EXERCISES

SKILL PRACTICE

In Exercises 1 and 2, use the diagram at the right.

1. VOCABULARY Explain what $MN$ means and what $MN$ means.

2. WRITING Explain how you can find $PN$ if you know $PQ$ and $QN$. How can you find $PN$ if you know $MP$ and $MN$?

EXAMPLE 1

Measure the length of the segment to the nearest tenth of a centimeter.

3. $A$ $B$

4. $C$ $D$

5. $E$

SEGMENT ADDITION POSTULATE Find the indicated length.

6. Find $MP$.

7. Find $RT$.

8. Find $UW$.


10. Find $BC$.

11. Find $DE$.

12. ERROR ANALYSIS In the figure at the right, $AC = 14$ and $AB = 9$. Describe and correct the error made in finding $BC$.

EXAMPLE 4

CONGRUENCE In Exercises 13–15, plot the given points in a coordinate plane. Then determine whether the line segments named are congruent.

13. $A(0, 1)$, $B(4, 1)$, $C(1, 2)$, $D(1, 6)$; $AB$ and $CD$

14. $J(-6, -8)$, $K(-6, 2)$, $L(-2, -4)$, $M(-6, -4)$; $JK$ and $LM$

15. $R(-200, 300)$, $S(200, 300)$, $T(300, -200)$, $U(300, 100)$; $RS$ and $TU$

ALGEBRA Use the number line to find the indicated distance.

16. $JK$

17. $JL$

18. $JM$

19. $KM$

20. SHORT RESPONSE Use the diagram. Is it possible to use the Segment Addition Postulate to show that $FB > CB$ or that $AC > DB$? Explain.
FINDING LENGTHS In the diagram, points V, W, X, Y, and Z are collinear, VZ = 52, XZ = 20, and WX = XY = YZ. Find the indicated length.

21. WX  
22. VW  
23. WY  
24. VX  
25. WZ  
26. VY

27. ★ MULTIPLE CHOICE Use the diagram.
What is the length of BE?

A. 1  
B. 4.4  
C. 10  
D. 16

قدر ג"א Point S is between R and T on RT. Use the given information to write an equation in terms of x. Solve the equation. Then find RS and ST.

28. RS = 2x + 10  
ST = x - 4  
RT = 21

29. RS = 3x - 16  
ST = 4x - 8  
RT = 60

30. RS = 2x - 8  
ST = 3x - 10  
RT = 17

31. CHALLENGE In the diagram, AB ≅ BC, AC ≅ CD, and AD = 12. Find the lengths of all the segments in the diagram. Suppose you choose one of the segments at random. What is the probability that the measure of the segment is greater than 3? Explain.

PROBLEM SOLVING

32. SCIENCE The photograph shows an insect called a walkingstick. Use the ruler to estimate the length of the abdomen and the length of the thorax to the nearest \( \frac{1}{4} \) inch. About how much longer is the walkingstick’s abdomen than its thorax?

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33. MODEL AIRPLANE In 2003, a remote-controlled model airplane became the first ever to fly nonstop across the Atlantic Ocean. The map shows the airplane’s position at three different points during its flight.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>A Leave Cape Spear, Newfoundland</td>
<td></td>
</tr>
<tr>
<td>B Approximate position</td>
<td></td>
</tr>
<tr>
<td>after about 1 day</td>
<td></td>
</tr>
<tr>
<td>C Land at Mannin Bay, Ireland, after nearly 38 hours</td>
<td></td>
</tr>
</tbody>
</table>

a. Find the total distance the model airplane flew.

b. The model airplane’s flight lasted nearly 38 hours. Estimate the airplane’s average speed in miles per hour.

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1.2 Use Segments and Congruence
34. **SHORT RESPONSE** The bar graph shows the win-loss record for a lacrosse team over a period of three years.

a. Use the scale to find the length of the yellow bar for each year. What does the length represent?

b. For each year, find the percent of games lost by the team.

c. *Explain* how you are applying the Segment Addition Postulate when you find information from a stacked bar graph like the one shown.

![Bar Graph]

![Number of games]

- Wins
- Losses

35. **MULTI-STEP PROBLEM** A climber uses a rope to descend a vertical cliff. Let A represent the point where the rope is secured at the top of the cliff, let B represent the climber’s position, and let C represent the point where the rope is secured at the bottom of the cliff.

a. **Model** Draw and label a line segment that represents the situation.

b. **Calculate** If AC is 52 feet and AB is 31 feet, how much farther must the climber descend to reach the bottom of the cliff?

36. **CHALLENGE** Four cities lie along a straight highway in this order: City A, City B, City C, and City D. The distance from City A to City B is 5 times the distance from City B to City C. The distance from City A to City D is 2 times the distance from City A to City B. Copy and complete the mileage chart.

<table>
<thead>
<tr>
<th></th>
<th>City A</th>
<th>City B</th>
<th>City C</th>
<th>City D</th>
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<tr>
<td>City A</td>
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<td>City D</td>
<td>?</td>
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</tr>
</tbody>
</table>

**Michigan Mixed Review**

37. Which function best describes the graph at the right?

- **A** \( f(x) = -\frac{2}{3} x + 2 \)
- **B** \( f(x) = -\frac{3}{2} x - 2 \)
- **C** \( f(x) = \frac{2}{3} x + 2 \)
- **D** \( f(x) = \frac{3}{2} x - 2 \)

38. Which coordinates represent a point that lies in Quadrant II?

- **A** (1, 5)
- **B** (−1, 5)
- **C** (1, −5)
- **D** (−1, −5)

**Extra Practice** for Lesson 1.2, p. 896 **Online Quiz** at classzone.com
1.3 **EXERCISES**

**SKILL PRACTICE**

1. **VOCABULARY** Copy and complete: To find the length of $AB$, with endpoints $A(-7, 5)$ and $B(4, -6)$, you can use the __________.

2. **WRITING** *Explain* what it means to bisect a segment. Why is it impossible to bisect a line?

**FINDING LENGTHS** Line $\ell$ bisects the segment. Find the indicated length.

3. Find $RT$ if $RS = 5\frac{1}{2}$ in.
4. Find $UW$ if $VW = \frac{5}{8}$ in.
5. Find $EG$ if $EF = 13$ cm.

6. Find $BC$ if $AC = 19$ cm.
7. Find $QR$ if $PR = 9\frac{1}{2}$ in.
8. Find $LM$ if $LN = 137$ mm.

9. **SEGMENT BISECTOR** Line $RS$ bisects $\overline{PQ}$ at point $R$. Find $RQ$ if $PQ = 4\frac{3}{4}$ inches.

10. **SEGMENT BISECTOR** Point $T$ bisects $\overline{UV}$. Find $UV$ if $UT = 2\frac{7}{8}$ inches.

**ALGEBRA** In each diagram, $M$ is the midpoint of the segment. Find the indicated length.

11. Find $AM$.
12. Find $EM$.
13. Find $JM$.

14. Find $PR$.
15. Find $SU$.
16. Find $XZ$.

**FINDING MIDPOINTS** Find the coordinates of the midpoint of the segment with the given endpoints.

17. $C(3, 5)$ and $D(7, 5)$
18. $E(0, 4)$ and $F(4, 3)$
19. $G(-4, 4)$ and $H(6, 4)$
20. $J(-7, -5)$ and $K(-3, 7)$
21. $P(-8, -7)$ and $Q(11, 5)$
22. $S(-3, 3)$ and $T(-8, 6)$
23. **WRITING** Develop a formula for finding the midpoint of a segment with endpoints $A(0, 0)$ and $B(m, n)$. *Explain* your thinking.

1.3 Use Midpoint and Distance Formulas 19
24. **ERROR ANALYSIS** Describe the error made in finding the coordinates of the midpoint of a segment with endpoints $S(8, 3)$ and $T(2, -1)$. \[rac{(8 - 2, 3 - (-1))}{2} = (3, 2) \times \]

**FINDING ENDPOINTS** Use the given endpoint $R$ and midpoint $M$ of $\overline{RS}$ to find the coordinates of the other endpoint $S$.

25. $R(3, 0), M(0, 5)$  
26. $R(5, 1), M(1, 4)$  
27. $R(6, -2), M(5, 3)$  
28. $R(-7, 11), M(2, 1)$  
29. $R(4, -6), M(-7, 8)$  
30. $R(-4, -6), M(3, -4)$

**EXAMPLE 4**

**DISTANCE FORMULA** Find the length of the segment. Round to the nearest tenth of a unit.

31.  
32.  
33.

34. **★ MULTIPLE CHOICE** The endpoints of $\overline{MN}$ are $M(-3, -9)$ and $N(4, 8)$. What is the approximate length of $\overline{MN}$?

- **A** 1.4 units  
- **B** 7.2 units  
- **C** 13 units  
- **D** 18.4 units

**NUMBER LINE** Find the length of the segment. Then find the coordinate of the midpoint of the segment.

35.  
36.  
37.  
38.  
39.  
40.

41. **★ MULTIPLE CHOICE** The endpoints of $\overline{LF}$ are $L(-2, 2)$ and $F(3, 1)$. The endpoints of $\overline{JR}$ are $J(1, -1)$ and $R(2, -3)$. What is the approximate difference in the lengths of the two segments?

- **A** 2.24  
- **B** 2.86  
- **C** 5.10  
- **D** 7.96

42. **★ SHORT RESPONSE** One endpoint of $\overline{PQ}$ is $P(-2, 4)$. The midpoint of $\overline{PQ}$ is $M(1, 0)$. Explain how to find $PQ$.

**COMPARING LENGTHS** The endpoints of two segments are given. Find each segment length. Tell whether the segments are congruent.

43. $\overline{AB}: A(0, 2), B(-3, 8)$  
44. $\overline{EF}: E(1, 4), F(5, 1)$  
45. $\overline{JK}: J(-4, 0), K(4, 8)$  

46. **★ ALGEBRA** Points $S$, $T$, and $P$ lie on a number line. Their coordinates are $0$, $1$, and $x$, respectively. Given $SP = PT$; what is the value of $x$?

47. **CHALLENGE** $M$ is the midpoint of $\overline{JK}$. $JM = \frac{x}{8}$ and $JK = \frac{3x}{4} - 6$. Find $MK$. 

○ = WORKED-OUT SOLUTIONS  
★ = STANDARDIZED TEST PRACTICE
48. **WINDMILL** In the photograph of a windmill, $ST$ bisects $QR$ at point $M$. The length of $QM$ is $18 \frac{1}{2}$ feet. Find $QR$ and $MR$.

49. **DISTANCES** A house and a school are 5.7 kilometers apart on the same straight road. The library is on the same road, halfway between the house and the school. Draw a sketch to represent this situation. Mark the locations of the house, school, and library. How far is the library from the house?

**ARCHEOLOGY** The points on the diagram show the positions of objects at an underwater archaeological site. Use the diagram for Exercises 50 and 51.

50. Find the distance between each pair of objects. Round to the nearest tenth of a meter if necessary.
   - a. $A$ and $B$
   - b. $B$ and $C$
   - c. $C$ and $D$
   - d. $A$ and $D$
   - e. $B$ and $D$
   - f. $A$ and $C$

51. Which two objects are closest to each other? Which two are farthest apart?

52. **WATER POLO** The diagram shows the positions of three players during part of a water polo match. Player $A$ throws the ball to Player $B$, who then throws it to Player $C$. How far did Player $A$ throw the ball? How far did Player $B$ throw the ball? How far would Player $A$ have thrown the ball if he had thrown it directly to Player $C$? Round all answers to the nearest tenth of a meter.
53. **EXTENDED RESPONSE** As shown, a path goes around a triangular park.

a. Find the distance around the park to the nearest yard.

b. A new path and a bridge are constructed from point Q to the midpoint M of PR. Find QM to the nearest yard.

c. A man jogs from P to Q to M to R to Q and back to P at an average speed of 150 yards per minute. About how many minutes does it take? Explain.

54. **CHALLENGE** AB bisects CD at point M, CD bisects AB at point M, and $AB = 4 \cdot CM$. Describe the relationship between AM and CD.

---

**MICHIGAN MIXED REVIEW**

55. What are the solutions to $x^2 + 4x = 5$?

- **A** $x = 1$ and $x = 5$
- **B** $x = 1$ and $x = -5$
- **C** $x = -1$ and $x = 5$
- **D** $x = -1$ and $x = -5$

56. Juan is putting up a wallpaper border along the top of each wall in his rectangular living room. The border costs $9.25 per roll plus 7.75% sales tax. One roll is 15 feet long. What other information is needed to determine the number of rolls of border he needs to purchase?

- **A** The perimeter of the room
- **B** The total cost of each roll of border
- **C** The weight of one roll of border
- **D** The height of the walls in the room

57. Sally can jog at a rate of 6.5 miles per hour. If she continued in a straight path at this rate, what distance would she travel in 24 minutes?

- **A** 2.2 miles
- **B** 2.4 miles
- **C** 2.6 miles
- **D** 2.8 miles

---

**QUIZ for Lessons 1.1–1.3**

1. Sketch two lines that intersect the same plane at two different points. The lines intersect each other at a point not in the plane. (p. 2)

In the diagram of collinear points, $AE = 26$, $AD = 15$, and $AB = BC = CD$. Find the indicated length. (p. 9)

2. $DE$

3. $AB$

4. $AC$

5. $BD$

6. $CE$

7. $BE$

8. The endpoints of $RS$ are $R(-2, -1)$ and $S(2, 3)$. Find the coordinates of the midpoint of $RS$. Then find the distance between $R$ and $S$. (p. 15)

---

**EXTRA PRACTICE** for Lesson 1.3, p. 896

**ONLINE QUIZ** at classzone.com
An angle bisector is a ray that divides an angle into two angles that are congruent. In the activity on page 27, $\overline{BD}$ bisects $\angle ABC$. So, $\angle ABD \equiv \angle DBC$ and $m\angle ABD = m\angle DBC$.

**Example 5** Double an angle measure

In the diagram at the right, $\overline{YW}$ bisects $\angle XYZ$, and $m\angle XYW = 18^\circ$. Find $m\angle XYZ$.

**Solution**

By the Angle Addition Postulate, $m\angle XYZ = m\angle XYW + m\angle WYZ$. Because $\overline{YW}$ bisects $\angle XYZ$, you know that $\angle XYW \equiv \angle WYZ$.

So, $m\angle XYW = m\angle WYZ$, and you can write

$$m\angle XYZ = m\angle XYW + m\angle WYZ = 18^\circ + 18^\circ = 36^\circ.$$ 

**Guided Practice** for Example 5

7. Angle $\overline{MNP}$ is a straight angle, and $\overrightarrow{NQ}$ bisects $\angle MNP$. Draw $\angle MNP$ and $\overrightarrow{NQ}$. Use arcs to mark the congruent angles in your diagram, and give the angle measures of these congruent angles.

### 1.4 Exercises

**Skill Practice**

1. **VOCABULARY** Sketch an example of each of the following types of angles: acute, obtuse, right, and straight.

2. **WRITING** Explain how to find the measure of $\angle PQR$, shown at the right.

**Example 1**

**NAMING ANGLES AND ANGLE PARTS** In Exercises 3–5, write three names for the angle shown. Then name the vertex and sides of the angle.

3. 

4. 

5. 

---

28 Chapter 1 Essentials of Geometry
6. **NAMING ANGLES** Name three different angles in the diagram at the right.

![Diagram with angles](image)

7. \( m\angle W = 180^\circ \)  
8. \( m\angle X = 30^\circ \)  
9. \( m\angle Y = 90^\circ \)  
10. \( m\angle Z = 95^\circ \)

**CLASSIFYING ANGLES** Classify the angle with the given measure as *acute*, *obtuse*, *right*, or *straight*.

11. \( \angle JFL \)  
12. \( \angle GFH \)  
13. \( \angle GFK \)  
14. \( \angle GFL \)

**MEASURING ANGLES** Trace the diagram and extend the rays. Use a protractor to find the measure of the given angle. Then classify the angle as *acute*, *obtuse*, *right*, or *straight*.

15. \( \angle ACB \)  
16. \( \angle ABC \)  
17. \( \angle BFD \)  
18. \( \angle AEC \)  
19. \( \angle BDC \)  
20. \( \angle BEC \)

**NAMING AND CLASSIFYING** Give another name for the angle in the diagram below. Tell whether the angle appears to be *acute*, *obtuse*, *right*, or *straight*.

21. **MULTIPLE CHOICE** Which is a correct name for the obtuse angle in the diagram?

\[ \begin{array}{ll}
(A) \quad \angle ACB & (B) \quad \angle ACD \\
(C) \quad \angle BCD & (D) \quad \angle C
\end{array} \]

**ANGLE ADDITION POSTULATE** Find the indicated angle measure.

22. \( m\angle QST = ? \)  
23. \( m\angle ADC = ? \)  
24. \( m\angle NPM = ? \)

\[ \begin{array}{llll}
& Q & R & T & S \\
& 52^\circ & 47^\circ & & S
\end{array} \]

\[ \begin{array}{llll}
& A & D & C & B \\
& 21^\circ & 44^\circ & &
\end{array} \]

\[ \begin{array}{llll}
& L & P & M & N \\
& & 79^\circ & &
\end{array} \]

**ALGEBRA** Use the given information to find the indicated angle measure.

25. Given \( m\angle WXZ = 80^\circ \), find \( m\angle YXZ \).

\[ \begin{array}{ll}
& W & X & Y & Z \\
& (x + 5)^\circ & (3x - 5)^\circ & &
\end{array} \]

26. Given \( m\angle FJH = 168^\circ \), find \( m\angle FJG \).

\[ \begin{array}{ll}
& F & G \\
& (6x - 15)^\circ & (x + 8)^\circ
\end{array} \]

27. **MULTIPLE CHOICE** In the diagram, the measure of \( \angle XYZ \) is 140°. What is the value of \( x \)?

\[ \begin{array}{ll}
(A) \quad 27 & (B) \quad 33 \\
(C) \quad 67 & (D) \quad 73
\end{array} \]

1.4 Measure and Classify Angles
28. **CONGRUENT ANGLES** In the photograph below, \( m \angle AED = 34^\circ \) and \( m \angle EAD = 112^\circ \). Identify the congruent angles in the diagram. Then find \( m \angle BDC \) and \( m \angle ADB \).

![Diagram](image)

29. **ANGLE BISECTORS** Given that \( \overline{WZ} \) bisects \( \angle XWY \), find the two angle measures not given in the diagram.

30.

31.

32. **ERROR ANALYSIS** \( \overline{KM} \) bisects \( \angle JKL \) and \( m \angle JKM = 30^\circ \). Describe and correct the error made in stating that \( m \angle JKL = 15^\circ \). Draw a sketch to support your answer.

33. \( a^\circ \)  
34. \( b^\circ \)

35. \( c^\circ \)  
36. \( d^\circ \)

37. \( e^\circ \)  
38. \( f^\circ \)

39. **ERROR ANALYSIS** A student states that \( \overline{AD} \) can bisect \( \angle AGC \). Describe and correct the student's error. Draw a sketch to support your answer.

40. **ALGEBRA** In each diagram, \( \overline{BD} \) bisects \( \angle ABC \). Find \( m \angle ABC \).

41.

42.

43. **SHORT RESPONSE** You are measuring \( \angle PQR \) with a protractor. When you line up \( \overline{QR} \) with the 20° mark, \( \overline{QP} \) lines up with the 80° mark. Then you move the protractor so that \( \overline{QR} \) lines up with the 15° mark. What mark does \( \overline{QP} \) line up with? Explain.

44. \( A(3, 3), B(0, 0), C(3, 0) \)  
45. \( A(-5, 4), B(1, 4), C(-2, -2) \)

46. \( A(-5, 2), B(-2, -2), C(4, -3) \)  
47. \( A(-3, -1), B(2, 1), C(6, -2) \)

\( \bigcirc = \text{WORKED-OUT SOLUTIONS} \) on p. WS1  
\( \bigstar = \text{STANDARDIZED TEST PRACTICE} \)
48. **ALGEBRA** Let \((2x - 12)^\circ\) represent the measure of an acute angle. What are the possible values of \(x\)?

49. **CHALLENGE** \(\overrightarrow{SQ}\) bisects \(\angle RST\), \(\overrightarrow{SP}\) bisects \(\angle RSQ\), and \(\overrightarrow{SV}\) bisects \(\angle RSP\). The measure of \(\angle VSP\) is 17°. Find \(m\angle TSQ\). Explain.

50. **FINDING MEASURES** In the diagram,

\[
m\angle AEB = \frac{1}{2} \cdot m\angle CED, \quad \text{and} \quad \angle AED
\]

is a straight angle. Find \(m\angle AEB\) and \(m\angle CED\).

---

**PROBLEM SOLVING**

51. **SCULPTURE** In the sculpture shown in the photograph, suppose the measure of \(\angle LMN\) is 78° and the measure of \(\angle PMN\) is 47°. What is the measure of \(\angle LMP\)?

---

52. **MAP** The map shows the intersection of three roads. Malcom Way intersects Sydney Street at an angle of 162°. Park Road intersects Sydney Street at an angle of 87°. Find the angle at which Malcom Way intersects Park Road.

---

**CONSTRUCTION** In Exercises 53–55, use the photograph of a roof truss.

53. In the roof truss, \(\overrightarrow{BC}\) bisects \(\angle ABC\) and \(\angle DEF\), \(m\angle ABC = 112^\circ\), and \(\angle ABC \equiv \angle DEF\). Find the measure of the following angles.

   a. \(m\angle DEF\)  
   b. \(m\angle ABG\)  
   c. \(m\angle CBG\)  
   d. \(m\angle DEG\)

54. In the roof truss, \(\overrightarrow{GB}\) bisects \(\angle DGF\). Find \(m\angle DGE\) and \(m\angle FGE\).

55. Name an example of each of the following types of angles: acute, obtuse, right, and straight.
GEOGRAPHY  For the given location on the map, estimate the measure of \( \angle PSL \), where \( P \) is on the Prime Meridian (0° longitude), \( S \) is the South Pole, and \( L \) is the location of the indicated research station.

56. Macquarie Island  57. Dumont d’Urville  58. McMurdo
59. Mawson  60. Syowa  61. Vostok

62. ★ EXTENDED RESPONSE  In the flag shown, \( \angle AFE \) is a straight angle and \( FC \) bisects \( \angle AFE \) and \( \angle BFD \).

a. Which angles are acute? obtuse? right?

b. Identify the congruent angles.

c. If \( m \angle AFB = 26^\circ \), find \( m \angle DFE \), \( m \angle BFC \), \( m \angle CFD \), \( m \angle AFC \), \( m \angle AFD \), and \( m \angle BFD \). Explain.

63. CHALLENGE  Create a set of data that could be represented by the circle graph at the right. Explain your reasoning.

---

MICHIGAN MIXED REVIEW

64. The equation \( y = 2.6x^2 - 3.4x + 1.2 \) shows the relationship between \( x \), the number of years since a company began business, and \( y \), the company’s profit in millions of dollars. What is the company’s profit after they are in business for 8 years?

A. $138 million  B. $140.4 million  C. $192.4 million  D. $194.8 million

65. A cylindrical salt shaker has a height of 7 centimeters and a diameter of 4 centimeters. Mike fills the salt shaker to 0.5 centimeter from the top. Which expression can be used to find the volume of salt in the salt shaker?

A. \( \pi(2^2)(7 - 0.5) \)  B. \( \pi(7 - 0.5)^2(2) \)  C. \( \pi(4^2)(7 - 0.5) \)  D. \( \pi(7 - 0.5)^2( \)
CONCEPT SUMMARY

Interpreting a Diagram

There are some things you can conclude from a diagram, and some you cannot. For example, here are some things that you can conclude from the diagram at the right:

- All points shown are coplanar.
- Points A, B, and C are collinear, and B is between A and C.
- \(\overrightarrow{AC}, \overrightarrow{BD},\) and \(\overrightarrow{BE}\) intersect at point B.
- \(\angle DBC\) and \(\angle EBC\) are adjacent angles, and \(\angle ABC\) is a straight angle.
- Point E lies in the interior of \(\angle DBC\).

In the diagram above, you cannot conclude that \(\overline{AB} \cong \overline{BC}\), that \(\angle DBE \cong \angle EBC\), or that \(\angle ABD\) is a right angle. This information must be indicated, as shown at the right.

1.5 EXERCISES

SKILL PRACTICE

1. **VOCABULARY** Sketch an example of adjacent angles that are complementary. Are all complementary angles adjacent angles? Explain.

2. **WRITING** Are all linear pairs supplementary angles? Are all supplementary angles linear pairs? Explain.

**EXAMPLE 1**

on p. 35
for Exs. 3–7

**IDENTIFYING ANGLES** Tell whether the indicated angles are adjacent.

3. \(\angle ABD\) and \(\angle DBC\)
4. \(\angle WXY\) and \(\angle XYZ\)
5. \(\angle LQM\) and \(\angle NQA\)

6. **IDENTIFYING ANGLES** Name a pair of complementary angles and a pair of supplementary angles.

7. 

---

38 Chapter 1 Essentials of Geometry
COMPLEMENTARY ANGLES $\angle 1$ and $\angle 2$ are complementary angles. Given the measure of $\angle 1$, find $m\angle 2$.

8. $m\angle 1 = 43^\circ$  
9. $m\angle 1 = 21^\circ$  
10. $m\angle 1 = 89^\circ$  
11. $m\angle 1 = 5^\circ$

SUPPLEMENTARY ANGLES $\angle 1$ and $\angle 2$ are supplementary angles. Given the measure of $\angle 1$, find $m\angle 2$.

12. $m\angle 1 = 60^\circ$  
13. $m\angle 1 = 155^\circ$  
14. $m\angle 1 = 130^\circ$  
15. $m\angle 1 = 27^\circ$

16. ★ MULTIPLE CHOICE The arm of a crossing gate moves $37^\circ$ from vertical. How many more degrees does the arm have to move so that it is horizontal?

A. $37^\circ$  
B. $53^\circ$  
C. $90^\circ$  
D. $143^\circ$

**EXAMPLE 3**

**EXAMPLE 4**

**EXAMPLE 5**

**EXTRA ALGEBRA** Find $m\angle DEG$ and $m\angle GEF$.

17. 

18. 

19. 

IDENTIFYING ANGLE PAIRS Use the diagram below. Tell whether the angles are vertical angles, a linear pair, or neither.

20. $\angle 1$ and $\angle 4$  
21. $\angle 1$ and $\angle 2$  
22. $\angle 3$ and $\angle 5$  
23. $\angle 2$ and $\angle 3$  
24. $\angle 7$, $\angle 8$, and $\angle 9$  
25. $\angle 5$ and $\angle 6$  
26. $\angle 6$ and $\angle 7$  
27. $\angle 5$ and $\angle 9$

28. ★ ALGEBRA Two angles form a linear pair. The measure of one angle is 4 times the measure of the other angle. Find the measure of each angle.

29. ERROR ANALYSIS Describe and correct the error made in finding the value of $x$.

30. ★ MULTIPLE CHOICE The measure of one angle is $24^\circ$ greater than the measure of its complement. What are the measures of the angles?

A. $24^\circ$ and $66^\circ$  
B. $24^\circ$ and $156^\circ$  
C. $33^\circ$ and $57^\circ$  
D. $78^\circ$ and $102^\circ$

**ALGEBRA** Find the values of $x$ and $y$.

31. 

32. 

33. 

1.5 Describe Angle Pair Relationships 39
REASONING Tell whether the statement is *always*, *sometimes*, or *never* true. *Explain* your reasoning.

34. An obtuse angle has a complement.
35. A straight angle has a complement.
36. An angle has a supplement.
37. The complement of an acute angle is an acute angle.
38. The supplement of an acute angle is an obtuse angle.

**FINDING ANGLES** $\angle A$ and $\angle B$ are complementary. Find $m\angle A$ and $m\angle B$.

39. $m\angle A = (3x + 2)^\circ$
   
   $m\angle B = (x - 4)^\circ$

40. $m\angle A = (15x + 3)^\circ$
   
   $m\angle B = (5x - 13)^\circ$

41. $m\angle A = (11x + 24)^\circ$
   
   $m\angle B = (x + 18)^\circ$

**FINDING ANGLES** $\angle A$ and $\angle B$ are supplementary. Find $m\angle A$ and $m\angle B$.

42. $m\angle A = (8x + 100)^\circ$

   $m\angle B = (2x + 50)^\circ$

43. $m\angle A = (2x - 20)^\circ$

   $m\angle B = (3x + 5)^\circ$

44. $m\angle A = (6x + 72)^\circ$

   $m\angle B = (2x + 28)^\circ$

45. **CHALLENGE** You are given that $\angle GHJ$ is a complement of $\angle RST$ and $\angle RST$ is a supplement of $\angle ABC$. Let $m\angle GHJ$ be $x^\circ$. What is the measure of $\angle ABC$? *Explain* your reasoning.

---

**PROBLEM SOLVING**

**IDENTIFYING ANGLES** Tell whether the two angles shown are *complementary*, *supplementary*, or *neither*.

46. [Clock images]

47. [Clock images]

48. [Clock images]

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**ARCHITECTURE** The photograph shows the Rock and Roll Hall of Fame in Cleveland, Ohio. Use the photograph to identify an example of the indicated type of angle pair.

49. Supplementary angles
50. Vertical angles
51. Linear pair
52. Adjacent angles

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53. **SHORT RESPONSE** Use the photograph shown at the right. Given that $\angle FGb$ and $\angle BGC$ are supplementary angles, and $m\angle FGb = 120^\circ$, *explain* how to find the measure of the complement of $\angle BGC$.

---

○ = WORKED-OUT SOLUTIONS on p. WS1

√ = STANDARDIZED TEST PRACTICE

○ = MULTIPL REPRESE
54. SHADOWS The length of a shadow changes as the sun rises. In the
diagram below, the length of CB is the length of a shadow. The end of the
shadow is the vertex of \( \angle ABC \), which is formed by the ground and the
sun's rays. Describe how the shadow and angle change as the sun rises.

55. \textbf{MULTIPLE REPRESENTATIONS} Let \( x \) be an angle measure. Let \( y_1 \) be
the measure of a complement of the angle and let \( y_2 \) be the measure of
a supplement of the angle.

\textbf{a. Writing an Equation} Write equations for \( y_1 \) as a function of \( x \), and for
\( y_2 \) as a function of \( x \). What is the domain of each function? \textit{Explain.}

\textbf{b. Drawing a Graph} Graph each function and \textit{describe} its range.

56. CHALLENGE The sum of the measures of two complementary angles
exceeds the difference of their measures by 86\(^\circ\). Find the measure of each
angle. \textit{Explain} how you found the angle measures.

---

**Michigan Mixed Review**

57. The point \((-1, y)\) is a solution of the equation \(6x + 5y = 19\).
What is the value of \( y \)?

\[ \begin{align*}
\text{A} & \quad -3 \\
\text{B} & \quad 1 \\
\text{C} & \quad 4 \\
\text{D} & \quad 5
\end{align*} \]

58. Anna swims diagonal laps in the pool shown.
About how many laps must she complete to swim
0.5 mile?

\[ \begin{align*}
\text{A} & \quad 73 \\
\text{B} & \quad 98 \\
\text{C} & \quad 127 \\
\text{D} & \quad 197
\end{align*} \]

---

**Quiz for Lessons 1.4–1.5**

In each diagram, \( \overline{BD} \) bisects \( \angle ABC \). Find \( m \angle ABD \) and \( m \angle DBC \). (p. 24)

\[ \begin{align*}
1. & \quad (x + 20)^\circ \\
2. & \quad (10x - 42)^\circ \\
3. & \quad (16x + 27)^\circ \\
\end{align*} \]

Find the measure of (a) the complement and (b) the supplement of \( \angle 1 \). (p. 35)

\[ \begin{align*}
4. & \quad m \angle 1 = 47^\circ \\
5. & \quad m \angle 1 = 19^\circ \\
6. & \quad m \angle 1 = 75^\circ \\
7. & \quad m \angle 1 = 2^\circ \\
\end{align*} \]
Example 3  Find side lengths

**ALGEBRA** A table is shaped like a regular hexagon. The expressions shown represent side lengths of the hexagonal table. Find the length of a side.

**Solution**

First, write and solve an equation to find the value of \( x \). Use the fact that the sides of a regular hexagon are congruent.

\[
3x + 6 = 4x - 2 \\
6 = x - 2 \\
3 = x
\]

Write equation.

Subtract 3x from each side.

Add 2 to each side.

Then find a side length. Evaluate one of the expressions when \( x = 8 \).

\[
3x + 6 = 3(8) + 6 = 30
\]

The length of a side of the table is 30 inches.

Guided Practice for Example 3

3. The expressions \( 8y^\circ \) and \( (9y - 15)^\circ \) represent the measures of two of the angles in the table in Example 3. Find the measure of an angle.

### 1.6 Exercises

**Skill Practice**

1. **VOCABULARY** *Explain* what is meant by the term \( n \)-gon.

2. **WRITING** Imagine that you can tie a string tightly around a polygon. If the polygon is convex, will the length of the string be equal to the distance around the polygon? What if the polygon is concave? *Explain*.

**IDENTIFYING POLYGONS** Tell whether the figure is a polygon. If it is not, explain why. If it is a polygon, tell whether it is convex or concave.

3. 4. 5. 6.

**MULTIPLE CHOICE** Which of the figures is a concave polygon?
CLASSIFYING  Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. Explain your reasoning.

8. [Diagram of a circle]

9. [Diagram of a hexagon with side lengths labeled 1 in.]

10. [Diagram of an equilateral triangle]

11. [Diagram of a triangle with sides labeled 5 m, 4 m, and 5 m]

12. [Diagram of a parallelogram]

13. [Diagram of a rectangle]

14. ERROR ANALYSIS  Two students were asked to draw a regular hexagon, as shown below. Describe the error made by each student.

Student A

[Diagram of a hexagon with unequal sides]

Student B

[Diagram of a hexagon with unequal angles]

15. ALGEBRA  The lengths (in inches) of two sides of a regular pentagon are represented by the expressions $5x - 27$ and $2x - 6$. Find the length of a side of the pentagon.

16. ALGEBRA  The expressions $(9x + 5)^\circ$ and $(11x - 25)^\circ$ represent the measures of two angles of a regular nonagon. Find the measure of an angle of the nonagon.

17. ALGEBRA  The expressions $3x - 9$ and $23 - 5x$ represent the lengths (in feet) of two sides of an equilateral triangle. Find the length of a side.

USING PROPERTIES  Tell whether the statement is always, sometimes, or never true.

18. A triangle is convex.

19. A decagon is regular.

20. A regular polygon is equiangular.

21. A circle is a polygon.

22. A polygon is a plane figure.

23. A concave polygon is regular.

DRAWING  Draw a figure that fits the description.

24. A triangle that is not regular

25. A concave quadrilateral

26. A pentagon that is equilateral but not equiangular

27. An octagon that is equiangular but not equilateral

ALGEBRA  Each figure is a regular polygon. Expressions are given for two side lengths. Find the value of $x$.

28. [Diagram of a square with side lengths labeled $x^2 + x$, $x^2 + 4$]

29. [Diagram of a hexagon with side lengths labeled $x^2 + 3x$, $x^2 + x + 2$]

30. [Diagram of an octagon with side lengths labeled $x^2 + 2x + 49$, $x^2 - x + 190$]
31. **Challenge** Regular pentagonal tiles and triangular tiles are arranged in the pattern shown. The pentagonal tiles are all the same size and shape and the triangular tiles are all the same size and shape. Find the angle measures of the triangular tiles. *Explain* your reasoning.

32. **Architecture** Longwood House, shown in the photograph on page 42, is located in Natchez, Mississippi. The diagram at the right shows the floor plan of a part of the house.
   a. Tell whether the red polygon in the diagram is convex or concave.
   b. Classify the red polygon and tell whether it appears to be regular.

33. **Example 2** Each sign suggests a polygon. Classify the polygon by the number of sides. Tell whether it appears to be equilateral, equiangular, or regular.

34. **Multiple Choice** Two vertices of a regular quadrilateral are $A(0, 4)$ and $B(0, -4)$. Which of the following could be the other two vertices?
   A. $C(4, 4)$ and $D(4, -4)$
   B. $C(-4, 4)$ and $D(-4, -4)$
   C. $C(8, -4)$ and $D(8, 4)$
   D. $C(0, 8)$ and $D(0, -8)$

35. **Multi-Step Problem** The diagram shows the design of a lattice made in China in 1850.
   a. Sketch five different polygons you see in the diagram. Classify each polygon by the number of sides.
   b. Tell whether each polygon you sketched is concave or convex, and whether the polygon appears to be equilateral, equiangular, or regular.
39. **SHORT RESPONSE** The shape of the button shown is a regular polygon. The button has a border made of silver wire. How many millimeters of silver wire are needed for this border? *Explain.*

40. **EXTENDED RESPONSE** A segment that joins two nonconsecutive vertices of a polygon is called a diagonal. For example, a quadrilateral has two diagonals, as shown below.

<table>
<thead>
<tr>
<th>Type of polygon</th>
<th>Diagram</th>
<th>Number of sides</th>
<th>Number of diagonals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadrilateral</td>
<td><img src="image" alt="Diagram" /></td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Pentagon</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Hexagon</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Heptagon</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

a. Copy and complete the table. *Describe* any patterns you see.

b. How many diagonals does an octagon have? a nonagon? *Explain.*

c. The expression \( \frac{n(n - 3)}{2} \) can be used to find the number of diagonals in an \( n \)-gon. Find the number of diagonals in a 60-gon.

41. **LINE SYMMETRY** A figure has *line symmetry* if it can be folded over exactly onto itself. The fold line is called the *line of symmetry.* A regular quadrilateral has four lines of symmetry, as shown. Find the number of lines of symmetry in each polygon.

   a. A regular triangle  
   b. A regular pentagon  
   c. A regular hexagon  
   d. A regular octagon

42. **CHALLENGE** The diagram shows four identical squares lying edge-to-edge. Sketch all the different ways you can arrange four squares edge-to-edge. Sketch all the different ways you can arrange five identical squares edge-to-edge.

---

**MICHIGAN MIXED REVIEW**

42. A function \( f(x) = 2x^2 + 1 \) has \( \{1, 3, 5, 8\} \) as the replacement set for the independent variable \( x \). Which of the following is contained in the corresponding set for the dependent variable?

   - **A** 0  
   - **B** 5  
   - **C** 15  
   - **D** 19

43. The radius of Cylinder A is three times the radius of Cylinder B. The heights of the cylinders are equal. How many times greater is the volume of Cylinder A than the volume of Cylinder B?

   - **A** 3  
   - **B** 6  
   - **C** 9  
   - **D** 27

---

**EXTRA PRACTICE** for Lesson 1.6, p. 897  
**ONLINE QUIZ** at classzone.com
**Example 5** Find unknown length

The base of a triangle is 28 meters. Its area is 308 square meters. Find the height of the triangle.

**Solution**

\[ A = \frac{1}{2}bh \]

Write formula for the area of a triangle.

\[ 308 = \frac{1}{2}(28)h \]

Substitute 308 for \(A\) and 28 for \(b\).

\[ 22 = h \]

Solve for \(h\).

\[ \text{The height is 22 meters.} \]

**Guided Practice** for Example 5

7. The area of a triangle is 64 square meters, and its height is 16 meters. Find the length of its base.

### 1.7 Exercises

**Skill Practice**

1. **Vocabulary** How are the diameter and radius of a circle related?

2. **Writing** Describe a real-world situation in which you would need to find a perimeter, and a situation in which you would need to find an area. What measurement units would you use in each situation?

3. **Error Analysis** Describe and correct the error made in finding the area of a triangle with a height of 9 feet and a base of 52 feet.

\[ A = 52(9) = 468 \text{ ft}^2 \]

**Perimeter and Area** Find the perimeter and area of the shaded figure.

4.  
   - Perimeter: \(18 + 8 + 18 + 8 = 52\) ft
   - Area: \(8 \times 18 = 144\) sq ft

5.  
   - Perimeter: \(4.2 + 7 + 4.2 + 7 = 22.4\) m
   - Area: \(7 \times 4.2 = 29.4\) sq m

6.  
   - Perimeter: \(15 + 15 + 15 + 15 = 60\) in
   - Area: \(15 \times 15 = 225\) sq in

7.  
   - Perimeter: \(72 + 30 + 78 + 72 = 262\) yd
   - Area: \(\frac{1}{2} \times 72 \times 78 = 2712\) sq yd

8.  
   - Perimeter: \(9 + 15 + 24 = 48\) mm
   - Area: \(\frac{1}{2} \times 15 \times 9 = 67.5\) sq mm

9.  
   - Perimeter: \(9 + 6 + 17 + 6 = 40\) cm
   - Area: \(\frac{1}{2} \times 9 \times 6 = 27\) sq cm

---

52 Chapter 1 Essentials of Geometry
10. **DRAWING A DIAGRAM** The base of a triangle is 32 feet. Its height is $16\frac{1}{2}$ feet. Sketch the triangle and find its area.

**CIRCUMFERENCE AND AREA** Use the given diameter $d$ or radius $r$ to find the circumference and area of the circle. Round to the nearest tenth.

11. $d = 27$ cm  
12. $d = 5$ in.  
13. $r = 12.1$ cm  
14. $r = 3.9$ cm

15. **DRAWING A DIAGRAM** The diameter of a circle is 18.9 centimeters. Sketch the circle and find its circumference and area. Round your answers to the nearest tenth.

**DISTANCE FORMULA** Find the perimeter of the figure. Round to the nearest tenth of a unit.

16.  
17.  
18.  

19. **MULTIPLE CHOICE** What is the approximate area (in square units) of the rectangle shown at the right?

(A) 6.7  
(B) 8.0  
(C) 9.0  
(D) 10.0

**CONVERTING UNITS** Copy and complete the statement.

20. $187 \text{ cm}^2 = \_ \_ \text{ m}^2$  
21. $13 \text{ ft}^2 = \_ \_ \text{ yd}^2$  
22. $18 \text{ in.}^2 = \_ \_ \text{ ft}^2$

23. $8 \text{ km}^2 = \_ \_ \text{ m}^2$  
24. $12 \text{ yd}^2 = \_ \_ \text{ ft}^2$  
25. $24 \text{ ft}^2 = \_ \_ \text{ in.}^2$

26. **MULTIPLE CHOICE** A triangle has an area of 2.25 square feet. What is the area of the triangle in square inches?

(A) 27 in.$^2$  
(B) 54 in.$^2$  
(C) 144 in.$^2$  
(D) 324 in.$^2$

**UNKNOWN MEASURES** Use the information about the figure to find the indicated measure.

27. Area = 261 m$^2$  
Find the height $h$.

28. Area = 66 in.$^2$  
Find the base $b$.

29. Perimeter = 25 in.  
Find the width $w$.
30. **UNKNOWN MEASURE** The width of a rectangle is 17 inches. Its perimeter is 102 inches. Find the length of the rectangle.

31. **ALGEBRA** The area of a rectangle is 18 square inches. The length of the rectangle is twice its width. Find the length and width of the rectangle.

32. **ALGEBRA** The area of a triangle is 27 square feet. Its height is three times the length of its base. Find the height and base of the triangle.

33. **ALGEBRA** Let \( x \) represent the side length of a square. Find a regular polygon with side length \( x \) whose perimeter is twice the perimeter of the square. Find a regular polygon with side length \( x \) whose perimeter is three times the length of the square. *Explain* your thinking.

**FINDING SIDE LENGTHS** Find the side length of the square with the given area. Write your answer as a radical in simplest form.

34. \( A = 184 \text{ cm}^2 \)  
35. \( A = 346 \text{ in.}^2 \)  
36. \( A = 1008 \text{ mi}^2 \)  
37. \( A = 1050 \text{ k} \)

38. **SHORT RESPONSE** In the diagram, the diameter of the yellow circle is half the diameter of the red circle. What fraction of the area of the red circle is not covered by the yellow circle? *Explain.*

39. **CHALLENGE** The area of a rectangle is 30 cm\(^2\) and its perimeter is 26 cm. Find the length and width of the rectangle.

### PROBLEM SOLVING

**EXAMPLES**

1 and 2 on pp. 49–50 for Exs. 40–41

**EXAMPLE 4** on p. 51 for Ex. 42

40. **WATER LILIES** The giant Amazon water lily has a lily pad that is shaped like a circle. Find the circumference and area of a lily pad with a diameter of 60 inches. Round your answers to the nearest tenth.

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41. **LAND** You are planting grass on a rectangular plot of land. You are also building a fence around the edge of the plot. The plot is 45 yards long and 30 yards wide. How much area do you need to cover with grass seed? How many feet of fencing do you need?

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42. **MULTI-STEP PROBLEM** Chris is installing a solar panel. The maximum amount of power the solar panel can generate in a day depends in part on its area. On a sunny day in the city where Chris lives, each square meter of the panel can generate up to 125 watts of power. The flat rectangular panel is 84 centimeters long and 54 centimeters wide.

a. Find the area of the solar panel in square meters.

b. What is the maximum amount of power (in watts) that the panel could generate if its area was 1 square meter? 2 square meters? *Explain.*

c. Estimate the maximum amount of power Chris's solar panel can generate. *Explain* your reasoning.
43. **MULTI-STEP PROBLEM** The eight spokes of a ship's wheel are joined at the wheel's center and pass through a large wooden circle, forming handles on the outside of the circle. From the wheel's center to the tip of the handle, each spoke is 21 inches long.

a. The circumference of the outer edge of the large wooden circle is 94 inches. Find the radius of the outer edge of the circle to the nearest inch.

b. Find the length \(x\) of a handle on the wheel. *Explain.*

44. **MULTIPLE REPRESENTATIONS** Let \(x\) represent the length of a side of a square. Let \(y_1\) and \(y_2\) represent the perimeter and area of that square.

a. **Making a Table** Copy and complete the table.

<table>
<thead>
<tr>
<th>Length, (x)</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter, (y_1)</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Area, (y_2)</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

b. **Making a Graph** Use the completed table to write two sets of ordered pairs: \((x, y_1)\) and \((x, y_2)\). Graph each set of ordered pairs.

c. **Analyzing Data** *Describe* any patterns you see in the table from part (a) and in the graphs from part (b).

45. **EXTENDED RESPONSE** The photograph at the right shows the Crown Fountain in Chicago, Illinois. At this fountain, images of faces appear on a large screen. The images are created by light-emitting diodes (LEDs) that are clustered in groups called modules. The LED modules are arranged in a rectangular grid.

a. The rectangular grid is approximately 7 meters wide and 15.2 meters high. Find the area of the grid.

b. Suppose an LED module is a square with a side length of 4 centimeters. How many rows and how many columns of LED modules would be needed to make the Crown Fountain screen? *Explain* your reasoning.

46. **ASTRONOMY** The diagram shows a gap in Saturn's circular rings. This gap is known as the *Cassini division.* In the diagram, the red circle represents the ring that borders the inside of the Cassini division. The yellow circle represents the ring that borders the outside of the division.

a. The radius of the red ring is 115,800 kilometers. The radius of the yellow ring is 120,600 kilometers. Find the circumference of the red ring and the circumference of the yellow ring. Round your answers to the nearest hundred kilometers.

b. Compare the circumferences of the two rings. About how many kilometers greater is the yellow ring's circumference than the red ring's circumference?
47. **CHALLENGE** In the diagram at the right, how many times as great is the area of the circle as the area of the square? *Explain* your reasoning.

48. **ALGEBRA** You have 30 yards of fencing with which to make a rectangular pen. Let \( x \) be the length of the pen.
   
   a. Write an expression for the width of the pen in terms of \( x \). Then write a formula for the area \( y \) of the pen in terms of \( x \).
   
   b. You want the pen to have the greatest possible area. What length and width should you use? *Explain* your reasoning.

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**MICHIGAN MIXED REVIEW**

49. Alexis is covering a Styrofoam ball with moss for a science fair project. She knows the radius of the ball and the number of square feet that one bag of moss will cover. Which formula should she use to determine the number of bags of moss needed?

   - **A** \( V = \frac{4}{3} \pi r^3 \)
   - **B** \( V = \frac{1}{3} \pi r^2 h \)
   - **C** \( S = 4 \pi r^2 \)
   - **D** \( S = 4 \pi rh \)

50. What are the coordinates of \( A \) after the translation \((x, y) \rightarrow (x - 1, y + 2)\)?

   - **A** \((0, 0)\)
   - **B** \((1, -3)\)
   - **C** \((-2, 0)\)
   - **D** \((-2, -4)\)

---

**QUIZ for Lessons 1.6–1.7**

Tell whether the figure is a polygon. If it is not, *explain* why. If it is a polygon, tell whether it is *convex or concave*. (p. 42)

1. 
2. 
3. 

Find the perimeter and area of the shaded figure. (p. 49)

4. 
5. 
6. 

7. **GARDENING** You are spreading wood chips on a rectangular garden. The garden is \(3 \frac{1}{2}\) yards long and \(2 \frac{1}{2}\) yards wide. One bag of wood chips covers 10 square feet. How many bags of wood chips do you need? (p. 49)
**BIG IDEAS**

**Describing Geometric Figures**
You learned to identify and classify geometric figures.

- **Point A**
- **Line AB (\(\overline{AB}\))**
- **Plane M**
- **Segment AB (\(\overline{AB}\))**
- **Ray AB (\(\overline{AB}\))**
- **Angle A** (\(\angle A, \angle BAC, \text{ or } \angle CAB\))
- **Polygon**
  - Quadrilateral \(ABCD\)
  - Pentagon \(PQRST\)

**Measuring Geometric Figures**

**SEGMENTS** You measured segments in the coordinate plane.

- **Distance Formula**
  
  Distance between \(A(x_1, y_1)\) and \(B(x_2, y_2)\):
  
  \[ AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \]

- **Midpoint Formula**
  
  Coordinates of midpoint \(M\) of \(\overline{AB}\), with endpoints \(A(x_1, y_1)\) and \(B(x_2, y_2)\):
  
  \[ M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \]

**ANGLES** You classified angles and found their measures.

- **Complementary angles**
  
  \(m\angle 1 + m\angle 2 = 90^\circ\)

- **Supplementary angles**
  
  \(m\angle 3 + m\angle 4 = 180^\circ\)

**FORMULAS** Perimeter and area formulas are reviewed on page 49.

**Understanding Equality and Congruence**

Congruent segments have equal lengths. Congruent angles have equal measures.

- \(\overline{AB} \cong \overline{BC}\) and \(\overline{AB} = \overline{BC}\)
- \(\angle JKL \cong \angle LKM\) and \(m\angle JKL = m\angle LKM\)
**REVIEW KEY VOCABULARY**

- undefined terms, p. 2
- point, line, plane
- collinear, coplanar points, p. 2
- defined terms, p. 3
- line segment, endpoints, p. 3
- ray, opposite rays, p. 3
- intersection, p. 4
- postulate, axiom, p. 9
- coordinate, p. 9
- distance, p. 9
- between, p. 10
- congruent segments, p. 11
- midpoint, p. 15
- segment bisector, p. 15
- angle, p. 24
- sides, vertex, measure
- acute, right, obtuse, straight, p. 25
- congruent angles, p. 26
- angle bisector, p. 28
- construction, p. 33
- complementary angles, p. 35
- supplementary angles, p. 3
- adjacent angles, p. 35
- linear pair, p. 37
- vertical angles, p. 37
- polygon, p. 42
- side, vertex
- convex, concave, p. 42
- n-gon, p. 43
- equilateral, equiangular, regular, p. 43

**VOCABULARY EXERCISES**

1. Copy and complete: Points $A$ and $B$ are the ___ of $\overline{AB}$.
2. Draw an example of a linear pair.
3. If $Q$ is between points $P$ and $R$ on $\overline{PR}$, and $PQ = QR$, then $Q$ is the ___ of $\overline{PR}$.

**REVIEW EXAMPLES AND EXERCISES**

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 1.

**1.1 Identify Points, Lines, and Planes**

**EXAMPLE**

Use the diagram shown at the right.

Another name for $\overrightarrow{CD}$ is line $m$.

Points $A$, $B$, and $C$ are collinear.

Points $A$, $B$, $C$, and $F$ are coplanar.

**EXERCISES**

4. Give another name for line $g$.
5. Name three points that are not collinear.
6. Name four points that are coplanar.
7. Name a pair of opposite rays.
8. Name the intersection of line $h$ and plane $M$. 
1.2 Use Segments and Congruence

**Example**

Find the length of $\overline{HJ}$.

$$GJ = GH + HJ \quad \text{Segment Addition Postulate}$$

$$27 = 18 + HJ \quad \text{Substitute 27 for GJ and 18 for GH.}$$

$$9 = HJ \quad \text{Subtract 18 from each side.}$$

**Exercises**

Find the indicated length.


10. Find $NP$.  

11. Find $XY$.

12. The endpoints of $\overline{DE}$ are $D(-4, 11)$ and $E(-4, -13)$. The endpoints of $\overline{GH}$ are $G(-14, 5)$ and $H(-9, 5)$. Are $\overline{DE}$ and $\overline{GH}$ congruent? Explain.

13 Use Midpoint and Distance Formulas

**Example**

$\overline{EF}$ has endpoints $E(1, 4)$ and $F(3, 2)$. Find (a) the length of $\overline{EF}$ rounded to the nearest tenth of a unit, and (b) the coordinates of the midpoint $M$ of $\overline{EF}$.

a. Use the Distance Formula.

$$EF = \sqrt{(3 - 1)^2 + (2 - 4)^2} = \sqrt{2^2 + (-2)^2} = \sqrt{8} \approx 2.8 \text{ units}$$

b. Use the Midpoint Formula.

$$M\left(\frac{1 + 3}{2}, \frac{4 + 2}{2}\right) = M(2, 3)$$

**Exercises**

13. Point $M$ is the midpoint of $\overline{JK}$. Find $JK$ when $JM = 6x - 7$ and $MK = 2x + 3$.

In Exercises 14–17, the endpoints of a segment are given. Find the length of the segment rounded to the nearest tenth. Then find the coordinates of the midpoint of the segment.

14. $A(2, 5)$ and $B(4, 3)$  

15. $F(1, 7)$ and $G(6, 0)$  

16. $H(-3, 9)$ and $J(5, 4)$  

17. $K(10, 6)$ and $L(0, -7)$  

18. Point $C(3, 8)$ is the midpoint of $\overline{AB}$. One endpoint is $A(-1, 5)$. Find the coordinates of endpoint $B$.

19. The endpoints of $\overline{EF}$ are $E(2, 3)$ and $F(8, 11)$. The midpoint of $\overline{EF}$ is $M$. Find the length of $\overline{EM}$.  

Chapter Review 61
1.4 Measure and Classify Angles

**EXAMPLE**

Given that \( m \angle YXV \) is 60°, find \( m \angle YXZ \) and \( m \angle ZXV \).

**STEP 1** Find the value of \( x \).

\[
    m \angle YXV = m \angle YXZ + m \angle ZXV
\]

\[
    60° = (2x + 11)° + (x + 13)°
\]

\[
    x = 12
\]

**Angle Addition Postulate**

**Substitute angle measures.**

**Solve for \( x \).**

**STEP 2** Evaluate the given expressions when \( x = 12 \).

\[
    m \angle YXZ = (2x + 11)° = (2 \cdot 12 + 11)° = 35°
\]

\[
    m \angle ZXV = (x + 13)° = (12 + 13)° = 25°
\]

**EXERCISES**

20. In the diagram shown at the right, \( m \angle LMN = 140° \). Find \( m \angle PMN \).

21. \( \overline{VZ} \) bisects \( \angle UVW \), and \( m \angle UVZ = 81° \). Find \( m \angle UVW \). Then classify \( \angle UVW \) by its angle measure.

1.5 Describe Angle Pair Relationships

**EXAMPLE**

a. \( \angle 1 \) and \( \angle 2 \) are complementary angles. Given that \( m \angle 1 = 37° \), find \( m \angle 2 \).

\[
    m \angle 2 = 90° - m \angle 1 = 90° - 37° = 53°
\]

b. \( \angle 3 \) and \( \angle 4 \) are supplementary angles. Given that \( m \angle 3 = 106° \), find \( m \angle 4 \).

\[
    m \angle 4 = 180° - m \angle 3 = 180° - 106° = 74°
\]

**EXERCISES**

\( \angle 1 \) and \( \angle 2 \) are complementary angles. Given the measure of \( \angle 1 \), find \( m \angle 2 \).

22. \( m \angle 1 = 12° \)

23. \( m \angle 1 = 83° \)

24. \( m \angle 1 = 46° \)

25. \( m \angle 1 = \)

\( \angle 3 \) and \( \angle 4 \) are supplementary angles. Given the measure of \( \angle 3 \), find \( m \angle 4 \).

26. \( m \angle 3 = 116° \)

27. \( m \angle 3 = 56° \)

28. \( m \angle 3 = 89° \)

29. \( m \angle 3 = \)

30. \( \angle 1 \) and \( \angle 2 \) are complementary angles. Find the measures of the angles when \( m \angle 1 = (x - 10)^° \) and \( m \angle 2 = (2x + 40)^° \).

31. \( \angle 1 \) and \( \angle 2 \) are supplementary angles. Find the measures of the angles when \( m \angle 1 = (3x + 50)^° \) and \( m \angle 2 = (4x + 32)^° \). Then classify \( \angle 1 \) by its angle measure.
1.6 Classify Polygons

**Example**

Classify the polygon by the number of sides. Tell whether it is equilateral, equiangular, or regular. *Explain.*

The polygon has four sides, so it is a quadrilateral. It is not equiangular or equilateral, so it is not regular.

**Exercises**

Classify the polygon by the number of sides. Tell whether it is equilateral, equiangular, or regular. *Explain.*

32. \[2 \text{ cm} \quad 2 \text{ cm} \quad 2 \text{ cm}\]

33. \[2.5 \text{ m} \quad 1 \text{ m} \quad 2.5 \text{ m} \quad 1 \text{ m}\]

34. 

35. Pentagon \(ABCDE\) is a regular polygon. The length of \(BC\) is represented by the expression \(5x - 4\). The length of \(DE\) is represented by the expression \(2x + 11\). Find the length of \(AB\).

1.7 Find Perimeter, Circumference, and Area

**Example**

The diameter of a circle is 10 feet. Find the circumference and area of the circle. Round to the nearest tenth.

The radius is half of the diameter, so \(r = \frac{1}{2}(10) = 5\) ft.

Circumference

\[C = 2\pi r = 2(3.14)(5) = 31.4 \text{ ft}\]

Area

\[A = \pi r^2 = 3.14(5^2) = 78.5 \text{ ft}^2\]

**Exercises**

In Exercises 36–38, find the perimeter (or circumference) and area of the figure described. If necessary, round to the nearest tenth.

36. Circle with diameter 15.6 meters

37. Rectangle with length \(4\frac{1}{2}\) inches and width \(2\frac{1}{2}\) inches

38. Triangle with vertices \(U(1, 2), V(−8, 2),\) and \(W(−4, 6)\)

39. The height of a triangle is 18.6 meters. Its area is 46.5 square meters. Find the length of the triangle's base.

40. The area of a circle is 320 square meters. Find the radius of the circle. Then find the circumference. Round your answers to the nearest tenth.
CHAPTER TEST

Use the diagram to decide whether the statement is true or false.

1. Point A lies on line m.
2. Point D lies on line n.
3. Points B, C, E, and Q are coplanar.
4. Points C, E, and B are collinear.
5. Another name for plane G is plane QEC.

Find the indicated length.

6. Find HJ.
7. Find BC.
8. Find XZ.

In Exercises 9–11, find the distance between the two points.

9. T(3, 4) and W(2, 7)
10. C(5, 10) and D(6, -1)
11. M(-8, 0) and N

12. The midpoint of AB is M(9, 7). One endpoint is A(3, 9). Find the coordinates of endpoint B.

13. Line t bisects CD at point M, CM = 3x, and MD = 27. Find CD.

In Exercises 14 and 15, use the diagram.

14. Trace the diagram and extend the rays. Use a protractor to measure ∠GHJ. Classify it as acute, obtuse, right, or straight.
15. Given m∠KHJ = 90°, find m∠LHJ.

16. The measure of ∠QRT is 154°, and RS bisects ∠QRT. What are the measures of ∠QRS and ∠SRT?

In Exercises 17 and 18, use the diagram at the right.

17. Name four linear pairs.
18. Name two pairs of vertical angles.

19. The measure of an angle is 64°. What is the measure of its complement? What is the measure of its supplement?

20. A convex polygon has half as many sides as a concave 10-gon. Draw the concave polygon and the convex polygon. Classify the convex polygon by the number of sides it has.

21. Find the perimeter of the regular pentagon shown at the right.

22. CARPET You can afford to spend $300 to carpet a room that is 5.5 yards long and 4.5 yards wide. The cost to purchase and install the carpet you like is $1.50 per square foot. Can you afford to buy this carpet? Explain.